

Notes - Properties of Logarithms

*Note that for all of these properties, the logarithms must have the same base!

Product Property: The logarithm of a product is the sum of the logarithms of its factors. (Add)

Examples: $\log_2 12 = \log_2 3 + \log_2 4$ OR $\log_5 6 + \log_5 x = \log_5 6x$

In a generalized symbolic form: $\log_b mn = \log_b m + \log_b n$

You try. Rewrite the following:

1. $\log_{12} 7 \cdot 3 =$

$\log_{12} 7 + \log_{12} 3$

2. $\ln 5 + \ln y =$

$\ln 5y$

Quotient Property: The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator. (Subtract)

Examples: $\log_3 \frac{5}{7} = \log_3 5 - \log_3 7$ OR $\log x - \log(2x - 3) = \log \frac{x}{2x - 3}$

In a generalized symbolic form: $\log_b \frac{m}{n} = \log_b m - \log_b n$

You try. Rewrite the following:

1. $\log_{12} \frac{7}{3} =$

$\log_{12} 7 - \log_{12} 3$

2. $\log 5x - \log 13$

$\log \frac{5x}{13}$

Power Property: The logarithm of a power is the product of the logarithm and the exponent. (Bring the exponent out front).

Examples: $\log_4 6^2 = 2\log_4 6$ OR $4\ln x = \ln x^4$

In a generalized symbolic form: $\log_b m^p = p\log_b m$

You try. Rewrite the following:

1. $\log_{15} 5^3 =$

$3\log_{15} 5$

2. $8\log_3 2 =$

$\log_3 2^8$

10-3 Skills Practice

Rewrite as a single logarithm

$$1. \log_6 3 + \log_6 4 = \log_6 12$$

$$2. \log 9 + \log C = \log 9C$$

$$3. \log_7 12 - \log_7 2 = \log_7 6$$

$$4. \ln 5 - \ln 2x = \ln \frac{5}{2x}$$

$$5. 3 \log_9 4 = \log_9 4^3 = \log_9 64$$

$$6. 4 \log_8 x + \log_8 3 = \log_8 3x^4$$

Write as separate logarithms

$$7. \ln \frac{3x}{7} = \ln 3x - \ln 7$$

$$8. \log_2 3x = \log_2 3 + \log_2 x$$

$$9. \ln 6 \cdot b^7 = \ln 6 + 7 \ln b$$

$$10. \log \frac{x^3}{5} = 3 \log x - \log 5$$

Solve each equation. Check your solutions.

$$11. \log_{10} 27 = 3 \log_{10} x = \log_{10} x^3$$

$$27 = x^3 \quad x = 3$$

$$12. 3 \log_7 4 = 2 \log_7 b$$

$$4^3 = b^2 \quad 64 = b^2 \quad b = 8$$

$$13. \log_4 5 + \log_4 x = \log_4 60$$

$$5x = 60 \quad x = 12$$

$$14. \log_6 2c + \log_6 8 = \log_6 80$$

$$16c = 80 \quad c = 5$$

$$15. \log_5 y - \log_5 8 = \log_5 1$$

$$\frac{y}{8} = 1 \quad y = 8$$

$$16. \log_2 q - \log_2 3 = \log_2 7$$

$$\frac{q}{3} = 7 \quad q = 21$$

$$17. \log_9 4 + 2 \log_9 5 = \log_9 w$$

$$4 \cdot 5^2 = w \quad w = 100$$

$$18. 3 \log_8 2 - \log_8 4 = \log_8 b$$

$$\frac{2^3}{4} = b \quad b = 2$$

$$19. \log_{10} x + \log_{10} (3x - 5) = \log_{10} 2$$

$$x(3x - 5) = 2 \quad x = \frac{5}{3}, 2$$

$$3x^2 - 5x - 2 = 0 \quad x = 2$$

$$20. \log_4 x + \log_4 (2x - 3) = \log_4 2$$

$$x(2x - 3) = 2$$

$$2x^2 - 3x - 2 = 0 \quad x = 2$$

$$21. \log_3 d + \log_3 3 = 3$$

$$\log_3 3d = 3 \quad 3^3 = 3d \quad 27 = 3d$$

$$d = 9$$

$$22. \log_{10} y - \log_{10} (2 - y) = 0$$

$$\log_{10} \frac{y}{2-y} = 0 \quad 1 = \frac{y}{2-y} \quad 2-y = y$$

$$2 = 2y \quad y = 1$$

$$23. \log_2 s + 2 \log_2 5 = 0$$

$$\log_2 25s = 0 \quad 1 = 25s \quad s = \frac{1}{25}$$

$$24. \log_2 (x + 4) - \log_2 (x - 3) = 3$$

$$\log_2 \frac{x+4}{x-3} = 3 \quad 8 = \frac{x+4}{x-3} \quad 8x - 24 = x + 4$$

$$7x = 28 \quad x = 4$$

$$25. \log_4 (n + 1) - \log_4 (n - 2) = 1$$

$$\log_4 \frac{n+1}{n-2} = 1 \quad 4 = \frac{n+1}{n-2} \quad 4n - 8 = n + 1$$

$$3n = 9 \quad n = 3$$

$$26. \log_5 10 + \log_5 12 = 3 \log_5 2 + \log_5 a$$

$$\log_5 120 = \log_5 8a$$

$$a = 15$$